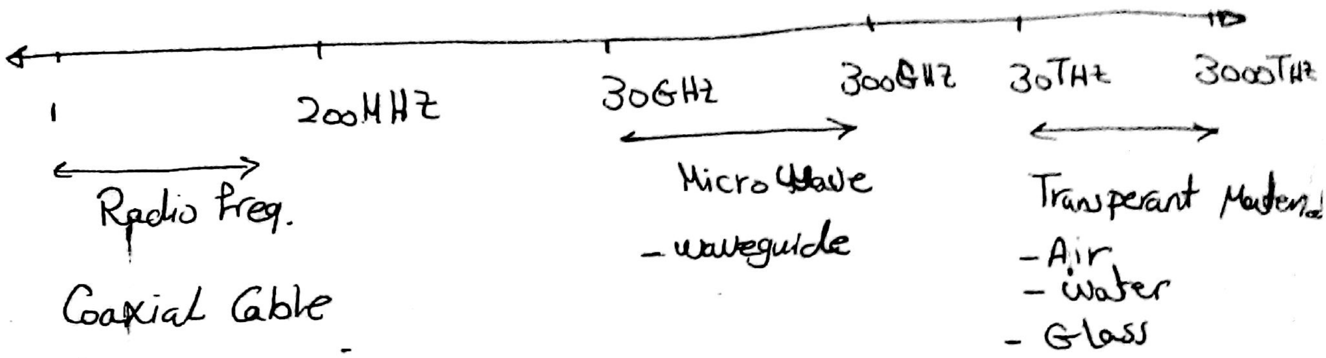
٤ إتصالات / optics / مرأية رضوان

Sec(1)

P

Optical Communication System

1) Spectrum of Light:



↳ There are 3-types of optoelectronic devices:

1) **Electro optic**: ~~device where~~ device where c/cs can be altered by Electric field.

↳ Characteristic like $\boxed{\varepsilon_r, \mu_r, n}$

↳ Ex. ~~polarizer~~ Modulator & Liquid Crystal

2) **Magneto-optic**: device where c/cs can be altered by Magnetic field.

↳ Ex: Polarizer

3) **Acousto-optic**: device where c/cs can be altered by Sound.

↳ Ex: Deflector

Note 1
ALL opto electronic devices based on Semi Conductor materials that have Conductivities within Range $\boxed{10^{-6} \text{ to } 10^{3}}$

Note 2
↳ Conductivity by semi Coductor Can be altered by:
1) Temperature
2) Illumenation
3) Doping.

Part (1) Study the electrical and optical properities of Semi Conductor [Evaluate energy levels Energy gap]

Some def.
↳ The wave Function $\Psi(x,t)$ ⇒ 1) Mathematical function that Contains all information on particle-waves
or
2) ↳ probability of finding the particle
↳ Things like energy and power are related to magnitude squared quantities:

* [illegible]

+ we say that the squared amplitude of the wave function gives the probability of finding the object at the particular point x, y, z and at specific time t

$$P(x,t) \propto |\Psi(x,t)|^2$$

* Note: Because the particle must be somewhere, the integration of $|\psi|^2$ over total space and time must be equal to 1

$$\int_{-\infty}^{\infty} |\psi|^2 dV = 1$$

↳ For 1D $\boxed{\int_{-\infty}^{\infty} |\psi|^2 dx = 1}$

↳ Normalization Condition.

Remember

1) Coulomb's Law: electrostatic attraction between electron and proton

$$F = K \cdot \frac{q_1 q_2}{r^2} = -\frac{Ke^2}{r^2}$$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$$

2) Potential Energy: work need to be done to bring an electron from the center of the atom to a distance r away from the center against this force

$$E_p = W = F \cdot r = \frac{-e^2}{4\pi\varepsilon_0 r^2} \cdot r$$

$$\boxed{E_p = \frac{-e^2}{4\pi\varepsilon_0 r}}$$

3) Plank's relation ⇒ $p = \frac{h}{\lambda}$

De-Broglie Relation → $mv = \frac{h}{\lambda}$ → $\boxed{\lambda = \frac{h}{mv}}$

Q(2) Derive the 1D time independant schrodinger eqn for isolated particle with mass m.

~Sol~ $E = E_K + E_P$

$E \rightarrow$ total Energy of Electron

$E_K \rightarrow$ Kinetic Energy

$U = E_P \rightarrow$ potential Energy

D Multiply with ψ

$\therefore \psi E = \psi E_K + \psi E_P$

$\psi E = \psi \cdot \frac{1}{2} m v^2 + \psi u$, but $P =$ momentum $= mv$

& $p^2 = m^2 v^2$

$\therefore \psi E = \psi \cdot \frac{p^2}{2m} + \psi u$

$\psi = e^{jKx} \longrightarrow \frac{d\psi}{dx} = jKe^{jKx} \longrightarrow \frac{d^2\psi}{dx^2} = -K^2 \cdot e^{jKx}$

$e^{jKx} = \psi$

From De-Broglie $\rightarrow$ $\boxed{P = \frac{h}{\lambda}}$ & $K = \frac{2\pi}{\lambda}$

Relation for particle-wave duality

$\therefore K = \frac{2\pi}{h} \cdot P = \frac{P}{\hbar}$

$\therefore \frac{d^2\psi}{dx^2} = -\frac{p^2}{\hbar^2}\psi \Rightarrow p^2\psi = -\hbar^2 \frac{d^2\psi}{dx^2}$

$$\therefore \boxed{\psi(E-u) + \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = 0} \rightarrow \text{Schrodinger Enq. in one Dim.}$$

Q (4) In a ~~region~~ region of Space, a particle with ~~mass~~ mass m and with Zero energy has a time independant wave Function $\psi(x) = A e^{-x^2/L^2}$, where A, L are Constants, determine the potential energy $U(x)$ of the particle?

~Sol~

Schrodinger eqn. $$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (E - U)\psi = 0$$

$$E = 0$$

$$\therefore \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - U\psi = 0$$

$$\rightarrow \boxed{\frac{d^2\psi}{dx^2} - \frac{2mU}{\hbar^2}\psi = 0} \rightarrow 1$$

$$\because \psi(x) = A e^{-x^2/L^2}$$

$$\frac{d\psi}{dx} = A\left(-\frac{2x}{L^2}\right) e^{-\frac{x^2}{L^2}}$$

$$\frac{d\psi}{dx} = -\left(\frac{2A}{L^2}\right) x e^{-\frac{x^2}{L^2}}$$

$$\frac{d^2\psi}{dx^2} = -\left(\frac{2A}{L^2}\right)\left[x\left(\frac{-2x}{L^2}\right)e^{-\frac{x^2}{L^2}} + e^{-\frac{x^2}{L^2}} \cdot 1\right]$$

$$= A e^{-x^2/L^2}\left[\frac{4x^2}{L^4} - \frac{2}{L^2}\right]$$

$$\frac{d^2\psi}{dx^2} = \psi\left(\frac{4x^2}{L^4} - \frac{2}{L^2}\right) \rightarrow \boxed{2}$$

From 1 & 2 ψ to be solution for 1

$$\therefore \frac{2m}{\hbar^2} U(x) = \left(\frac{4x^2}{L^4} - \frac{2}{L^2}\right)$$

$$U(x) = \frac{\cancel{4}^{2}\hbar^2}{\cancel{2}m L^4}\left(x^2 - \frac{L^2}{2}\right) \Rightarrow U(x) = \frac{2\hbar^2}{mL^4}\left(x^2 - \right.$$

Q(5) A free electron has a wave function $\psi(x) = \sin(kx - \omega t)$, determine the de-Broglie wavelength, momentum, kinetic energy and the speed of the electron when $k = 50\ nm^{-1}$.

~Sol~

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (E - u)\psi = 0$$

$\because$ Free electron $\Rightarrow \therefore U = $ Potential electron $= 0$

$$\therefore \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + E\psi = 0 \qquad * \frac{2m}{\hbar^2}$$

$$\therefore \boxed{\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0} \rightarrow (1)$$

$\because \psi = \sin(kx - \omega t)$

$$\frac{d\psi}{dx} = -k\cos(kx - \omega t)$$

$$\frac{d^2\psi}{dx^2} = -k^2\sin(kx - \omega t)$$

$$\frac{d^2\psi}{dx^2} = -k^2\psi \rightarrow (3)$$

From (3) ψ will be solution of Schrodinger eqn. when $\boxed{k^2 = \frac{2mE}{\hbar^2}}$

$$\because E = E_k = \frac{1}{2}m v^2 = \frac{P^2}{2m}$$

$$\therefore K^2 = \frac{\cancel{2m}\, P^2}{\hbar^2\, \cancel{2m}} = \frac{P^2}{\hbar^2}$$

$$\therefore P = \hbar \cdot K = \frac{h}{2\pi} \cdot K = \frac{6.626 \times 10^{-34} \times 50}{2\pi \cdot 10^{-9}}$$

$$\therefore \boxed{P = 5.27 \times 10^{-24}} \text{ Kg.m/s}$$

$$\therefore P = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{P} = \frac{h}{\frac{h}{2\pi} \cdot K} = \frac{2\pi}{K}$$

$$\boxed{\lambda = 0.125 nm} = 126 \text{ Pm}$$

$$\boxed{E_K = \frac{P^2}{2m} = 1.53 \times 10^{-17} \text{ Joule}}$$

$$= 95.4 \text{ eV}$$

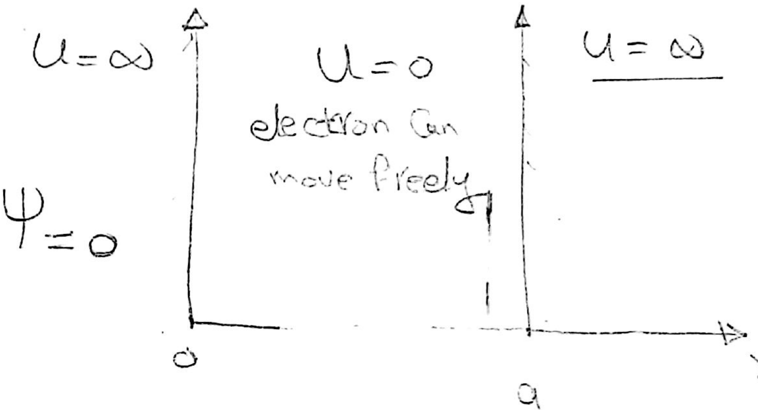
$$1 eV = 1.6022 \times 10^{-19} J$$

Q(6) Solve the one dimensional time independant Schrödinger for particle in a box, where $U(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$

(a) Write down a relation for the energy values and draw it with explanation

(b) Determine the wave function $\psi(x)$

~Sol~



$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (E-U)\psi = 0$$

at $U=0$

$$\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + E\psi = 0 \qquad * \frac{2m}{\hbar^2}$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \longrightarrow ①$$

Let $K = \frac{\sqrt{2mE}}{\hbar} \longrightarrow ②$

$$\therefore \frac{d^2\psi}{dx^2} + K^2\psi = 0$$

General solution for this differential eqn.

$$\psi(x) = A\sin(Kx) + B\cos(Kx)$$

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The particle can't exist outside the box ;

$\psi \rightarrow$ outside the box $= 0$

$\therefore$ we have 2-boundary condition

$\psi = 0$ at $x = 0$ | $\psi = 0$ at $x = a$

$\therefore 0 = A \sin(0) + B \cos(0)$

$\boxed{B = 0}$

$0 = A \sin(Ka)$

$Ka = n\pi$

$n = 1, 2, 3, 4, \dots$

$\therefore \boxed{K = \frac{n\pi}{a}}$

الـ (n) غير مسموح ان تكون بصفر لانه عند $n=0 \leftarrow K=0 \leftarrow \psi=0 \leftarrow$ معناه ان الـ electron غير موجود في اي مكان

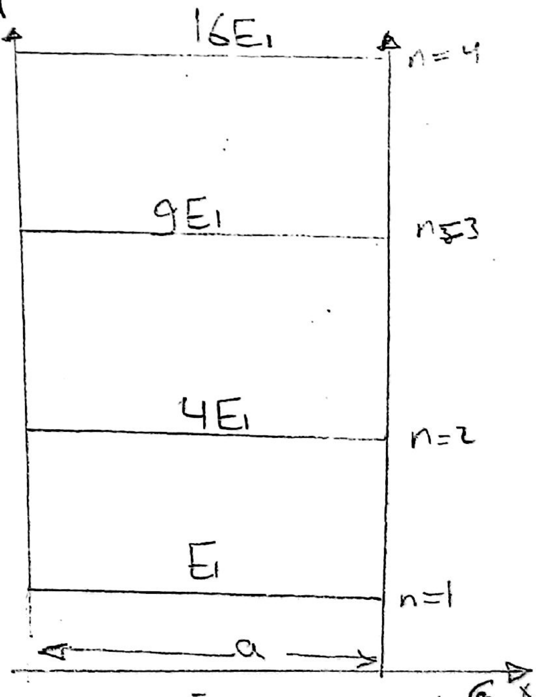
$$\therefore \frac{\sqrt{2mE}}{\hbar} = \frac{n\pi}{a} \Rightarrow E = \frac{n^2 \pi^2 \hbar^2}{2a^2 m}$$

$$\therefore E = \frac{n^2 \cancel{\pi^2}}{2a^2 m} \cdot \frac{h^2}{4\cancel{\pi^2}} = \frac{n^2 h^2}{8a^2 m}$$

$n = 1, 2, 3, 4, \dots$

$\therefore$ energy is quantaized inside the Box and zero is not allowed

$E_1 = \frac{h^2}{8a^2 m}$ at $n = 1$



$n=2$

$E_2 = 4E_1$

$n=3$

$E_3 = 9E_1$

$n=4$

$E_4 = 16E_1$

(b) For Determining ψ:

↔ To find A $\qquad \psi = A \sin\left(\frac{n\pi}{a} x\right)$

In general, The probability that the particle will be found Some where is 1

In our Case, the probability of finding electrom somewhere in the Wall is 1

$$\therefore \int_{x=0}^{x=a} |\psi|^2 dx = 1$$

$$\int_0^a A^2 \sin^2\left(\frac{n\pi}{a}x\right) = 1$$

$$\frac{A^2}{2}\int_0^a \left(1 - \cos\left(\frac{2n\pi}{a}x\right)\right)dx = 1$$

$$\frac{A^2}{2}\left[a - \frac{\overset{0}{\cancel{\sin\left(\frac{2n\pi}{a}a\right)}}}{\frac{2n\pi}{a}} + 0 + \overset{0}{\cancel{\sin(0)}}\right] = 1$$

$$A^2 = \frac{2}{a} \rightarrow \boxed{A = \sqrt{\frac{2}{a}}}$$

$$\therefore \boxed{\psi = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)}$$